EXTENDED ABSTRACT

Title: Applications of fuzzy metrics in econometrics

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Abstract

Lofti A. Zadeth (1965) introduced the notion of a diffuse set, namely, if X is a non-empty set, a correspondence is established between each element of X and a real number in the range]0,1]. In general, economic concepts are defined on sets where the boundary is perfectly defined, for example belonging to a region or a country, nevertheless there are many notions that cannot be defined in this way because their boundaries are not determined. For instance, the pollution, well being, etc. Therefore, this concept eliminates the notion of border to establish a degree of belonging to set X. To define a metric that allows us to measure the distance between two points using fuzzy sets, several authors have investigated the concept of fuzzy metric space, in particular, we follow the concept that was introduced by George and Veramani (1994). Such metric is enough for defining a topology by means of the basis of neighborhoods that is given by open balls. These notions are used to introduce new techniques into the field of econometrics. The first approach goes to provide

similarity indices and new distances. The advantage of this methodology is that under certain conditions, the product of fuzzy metrics remains a fuzzy metric. In this sense, we can model similarity indices that are in turn the product of other indices, preserving the metric structure, and providing the model with a simple way to understand the structure provided. We use this topological properties to characterize the elements under study. Spatial econometrics has played and continues to play an important role in the study of socio-economic problems within econometrics. The concept of spatial dependence are related with spatial spillovers derived from concentration effects and omitted explanatory variables. In these methodologies, the weight matrix W was introduced. Following the same procedure used in the construction of binary matrices using Euclidean distance as the conceptual framework, we introduce another approach with the use of fuzzy and pseudo metrics. In order to show these framework, we provide and example in order to analyses the spatial patterns of innovation that relate technology transfer to imports trade in goods, business demography and research and development (R&D) expenditure in the 27 countries of the European Union (EU) from 2008 to 2015.

1 Introduction

In this work we introduce a new approach in the weighted matrices using fuzzy metrics. In order to show this tool we present an example that relate technology transfer to imports trade in goods, business demography and research and development (R&D) expenditure in the 27 countries of the European Union (EU) from 2008 to 2015, without Croatia where the time series not includes this country since 2008. Many authors has studied the external knowledge how the key input for innovation activities which performance different ways to model the dependence structure. For instance, Cabrer et al. (2007) define a proximity notion using the intensity in the commercial relationships interdependences between regions in order to explain the distribution of innovation activity. Jaffe et al. (1993) found evidence that proximity to innovation centers generates knowledge diffusion among companies, emphasizing the geographical relationship of patent citations. Selim Hazir et al. (2014) expressed the weight matrix as a convex combination of spatially proximate regions that are collaboration partners, not collaboration partners and distant collaboration partners to show the dependence structure. We define a proximity notion using the similarity with the patents subfields in order to explain the distribution of the EU internal trade. We select an appropriate strategic partners using fuzzy metrics in order to examines the relationship between number of patent applications to the EPO by priority year and the total R&D personnel and the total number of Intramural R&D expenditure into the EU countries in the 2008-2015 time period. Bottazzi et al. (2002) use the patent data in order to estimate the effect of R&D externalities in generating innovation. In order to analyze these externalities we asume that they depend on the distance between the regions, but also on the distance defined by externalities and on the distance defined by the externalities that are related. For instance, Finland and France are so far, nevertheless there could exist a dependence between together due to the two regions have hight correlation among the sectors where they have patents. Jaffe (1986) pointed out pure knowledge spillovers have been defined by looking at the technological similarity between sectors. Even though the technology allows the diffusion of knowledge through flows that are proximity in terms of neighborhoods, in our opinion, it is necessary take into consideration that spillover effects can be generated for reasons other than geographical ones. Therefore, the interest to establish a fuzzy metric is because it allows to measure distances in any sense and therefore establish an order for the locations in function to the established metric. Nowdays, the technology allows for a diffusion of knowledge through flows that are not proximity in terms of neighborhoods. Therefore it is necessary take into consideration that spillover effects can be generated for reasons other than geographical ones. The interest to establish a fuzzy metric is because it allows to measure distances in any sense and therefore establish an order for the locations in function to the established metric.

Let us introduce some technical formal concepts and definitions using the standard mathematical notation. A *t-norm* is a binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ which

satisfies for all elements $a,b,c,d \in [0,1]$: (1) the commutativity ($*(a,b) = *(b,a)$), (2) monotonicity ($*(a,b) \leq *(c,d)$ if $a \leq c$ and $b \leq d$), (3) associativity ($*(a,*(b,c)) = *(*(a,b),c)$) and (4) the number 1 acts as identity element. A *t-norm* is continuous if it is continuous as a function. We will denoted by $*$ a continuous t-norm in the usual topology on $[0,1] \times [0,1]$. Some classical examples used in fuzzy logic are (a) the usual product that we will denoted by $\cdot$, (b) the *minimum t-norm* denoted by $\wedge$ and the (c) *Lukasievicz t-norm* ($a * b = max\{a+b-1,0\}$ denoted by ℓ. These t-norms satisfies $a \wedge b \geq a \cdot b \geq \ell(a,b)$.
Let X be a non-empty set and M a membership function, that is, a function from X to $[0,1]$. A fuzzy set M is a pair (X,M) where the value $M(x)$ is called the membership of x in (X,M). When $M(x) = 0$ then we will say that x is not included in the fuzzy set (X,M). In a similar way, if $M(x) = 1$ (resp. $0 < M(x) < 1$) we say that x is fully (resp. partially) included. This definition is interesting in the field of Economy because there exists notions for which boundaries are not defined and therefore it is not possible to attribute membership or not to the set. For instance, the pollution, diseases, storms. We use the notion of fuzzy metric space corresponding to George and Veeramani (1994). A fuzzy metric space is an ordered triple $(X,M,*)$, where M is a fuzzy set on $X \times X \times]0,\infty] \to]0,1]$ satisfying the following conditions. For all $x,y,z \in X$, and $s,t > 0$:

1. $M(x,y,t) > 0$;
2. $M(x,y,t) = 1$ if and only if $x = y$.
3. $M(x,y,t) = M(y,x,t)$;
4. $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$;
5. $M(x,y,\cdot) :]0,+\infty] \to]0,1]$ is continuous.

Such a metric is enough for defining a topology τ_M by means of the basis of neighborhoods that is given by open balls $B_M(x,\varepsilon,t) := \{y \in X, 0 < r < 1, t > 0, M(x,y,t) > 1-r\}$. $M(x,y,t)$ may be interpreted as the degree of proximity between x and y with respect to the variable t. When a fuzzy metric M on X does not depend on t is said to be stationary, i.e. if for each $x,y \in X$ the function $M_{x,y}(t) = M(x,y,t)$ is constant.

The main methodological idea is that we can provide a weighted matrix using fuzzy metrics in order to capture any kind of relation. To show this framework, we consider a balanced panel data obtained from a data set where the individual indexes ($i = 1, ..., 27$) corresponds to the 27 EU countries and time ($t = 1, ..., 8$) over the period 2008-2015. The structure of the interactions between each pair of locations is represented by the fuzzy notions introduced in this work. The non zero elements of the matrix indicate whether two locations are neighbors in the sense of fuzzy notion. Therefore, each element w_{ij} represents the intensity relationship between the locations i and j. The questions and issues analysed in this work suggest that the patents into a given Member State depends not only the total business demography and the intramural R&D expenditure, but also the innovation similarity with another regions.

Through several neighborhood criteria we capture and measure different concepts and provide robust results to this analysis. For instance, fuzzy metrics have been used to created spatial weighted matrices, see Alama et al. 2016, in order to capture the measure of geographic proximity between FDI host countries, the measure of the similarity of the level of public debt between FDI host countries, and also a combined effect between the geographic proximity and the public debt.

The following tools are able to construct a matrix with the indices of points belonging to the set of the k nearest neighbours of each other given circle distances defined by the fuzzy metric. First of all, we use a weight matrix which capture the fuzzy proximity. We consider a fuzzy metric space $(X, M, *)$. In order to simplify we assume that it is stationary. For each location i, the function $M_i(j) = M(i, j)$ shows the fuzzy distance from i to j. We can order all locations with respect to this metric and the location i and select the k nearest locations, i.e. the k locations such that $M(i, j)$ is close to 1. Thus for each location i, we take 1 if the j location belongs into the k nearest neighbours and 0 otherwise. Therefore, from a binary k nearest neighbours list, in which regions are either listed as fuzzy neighbours or are absent to this, we standardised by rows, and the diagonal elements of the matrix are set to zero. For instance, the Figure 1 shows the spatial connectivity if we consider the four nearest neighbours using a fuzzy distance 1.

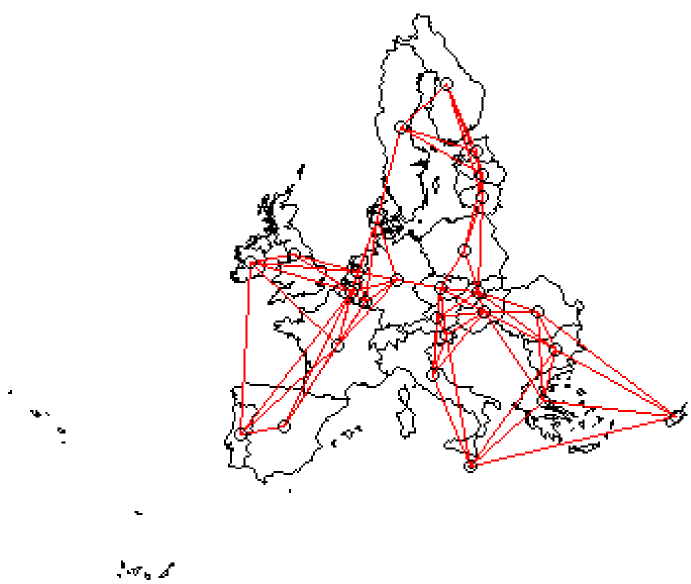

Figure 1: 4-NEAREST NEIGHBORS SPATIAL CONECTIVITY

In this work, we take three weight matrix. the first one has been built using the inverse of the euclidean distance,

$$D(i,j)_d = \frac{M/2}{M/2+dist(i,j)} \tag{1}$$

where M represents the maximum euclidean distance among all distances provinces $d(i,j)$ and the set X is defined by the coordinates of each location. For each location i, the function $D_i(j) = D(i,j)$ shows the fuzzy distance from i to j, if $w_i(j) = w(i,j)$ is close to 1 then the two locations are nearby. We call D the binary 6 nearest neighbours list, in which regions are either listed as fuzzy neighbours or are absent to this standardised by rows, and the diagonal elements of the matrix are set to zero.

The second one is build using a similarity fuzzy metric. Deltas et al. (2013) introduce a similarity index of research activities in regions i and j over k different productive sectors. They point out that the spillovers generated by the diffusion of knowledge depends on the similarity of research activities by the originator and recipient of the knowledge. We use patent sectors applications to the EPO by priority year by NUTS countries that we denote by P_{ik} measured in number of patents, provided by Eurostat database, in 27 UE countries over the period 2008-2015. Let X_k $k = 1,...,8$ be the set defined by patents granted by number in 8 sectors (human necessities; Performing operations and transporting; chemistry and metalurgy; textiles, paper; fixed constructions; mechanical engineering, lighting, heating, weapons; physics; electricity). Patents reflect inventive activity and they also show the capacity to exploit knowledge and translate it into potential economic gains. In this context, indicators based on patent statistics are widely used to assess the inventive performance of countries or regions. The grounds for the assumption that a patent represents a codification of inventive activity rely on the novelty, utility and inventiveness that an invention requires to be patented. For each productive sector we define a stationary fuzzy metrics defined by:

$$S_k(i,j) = \frac{min\{P_{ik},P_{jk}\}+0.1}{max\{P_{ik},P_{jk}\}+0.1} \tag{2}$$

All of them are fuzzy metrics under the same continuous t-norm $\cdot$ (usual product), then the product $S = (S_1 \times ... \times S_8, M, \cdot)$ is a fuzzy metric space under $\cdot$, see Segi et al. (2018).

We can define

$$S(i,j) = \prod_k \frac{min\{P_{ik}, P_{jk}\} + k}{max\{P_{ik}, P_{jk}\} + k} \tag{3}$$

Following the same procedure to the above fuzzy metric. We can order all locations with respect to this metric and the location i and select the k nearest locations, i.e. the k locations such that $S(i,j)$ is close to 1. Therefore, from a binary k nearest neighbours list, in which regions are either listed as fuzzy neighbours or are absent to this, we standardised by rows, and the diagonal elements of the matrix are set to zero. We call to this standarized weight matrix S.

Finally we can define a new weight matrix that summarize the distance and similarity notion by the product obtaining a new fuzzy metric under the t-continuous norm $\cdot$.

$$P(i,j) = D(i,j) * S(i,j) \tag{4}$$

In a similar way, we standarize matrix P simultaneously captures neighbourhood and similarity between regions.

2 Conclusion

In this work, we present a new tool in order to capture spatial dependence. The fuzzy theory provides the framework to construct weight matrices to express the interaction among regions and enabling to understand dependence among their economic performances. We show an example that relate technology transfer to imports trade in goods, business demography and research and development (R&D) expenditure in the 27 countries of the European Union (EU) from 2008 to 2015. We estimation results reveal that the patent sector interaction and the patent sector interaction mixed with distance among UE countries is key for understanding dependence among their patenting performance.

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