PAPER

Title: Migrants looking for opportunities - Putting back mass in the RUM based gravity equation for migration

Authors and e-mails of them: Damiaan Persyn (damiaan.persyn@uni-goettingen.de); Liesbeth Colen (liesbeth.colen@uni-goettingen.de)

Department: Department of Agricultural Economics and Rural Development

University: University of Göttingen, Germany

Subject area: *migration, discrete choice, urbanization, gravity equations*

Abstract: *This paper considers a random utility model (RUM) for migration where destina-tions differ in the number of opportunities, such as jobs or houses, contained in them. Under appropriate assumptions, this number of opportunities appears as the mass variable for the destination in the gravity equation for aggregate migration flows. In the literature, random utility based gravity models for migration frequently consider entire countries or regions as the atomistic unit of choice of migrants, and therefore lack a mass or size variable for the destination when predicting aggregate flows. Ignoring the size of the implicit choice set in migration destinations and the resulting omission of the destination mass in the gravity equation is problematic if destinations differ significantly in size, when interpreting residuals or estimated fixed effects, when interpreting coefficients on destination-specific variables which are correlated with size, or when predicting flows and the effect of policies or events thereon. We discuss several studies that may be affected by such bias. The proposed model leads to a set of recommendations for consistent empirical estimation of migration flows.*

Keywords: *migration, discrete choice, urbanisation, gravity equations*

JEL codes: O15, R23, F22

Migrants looking for opportunities - Putting back mass in the RUM based gravity equation for migration

Liesbeth Colen* Damiaan Persyn*
DARE, University of Göttingen

October 4, 2021
preliminary and incomplete — please do not cite

Abstract

This paper considers a random utility model (RUM) for migration where destinations differ in the number of opportunities, such as jobs or houses, contained in them. Under appropriate assumptions, this number of opportunities appears as the mass variable for the destination in the gravity equation for aggregate migration flows. In the literature, random utility based gravity models for migration frequently consider entire countries or regions as the atomistic unit of choice of migrants, and therefore lack a mass or size variable for the destination when predicting aggregate flows. Ignoring the size of the implicit choice set in migration destinations and the resulting omission of the destination mass in the gravity equation is problematic if destinations differ significantly in size, when interpreting residuals or estimated fixed effects, when interpreting coefficients on destination-specific variables which are correlated with size, or when predicting flows and the effect of policies or events thereon. We discuss several studies that may be affected by such bias. The proposed model leads to a set of recommendations for consistent empirical estimation of migration flows.

1 Introduction

The random utility based gravity equation used in current economic research on migration (see for example Beine *et al.*, 2016, for an overview) does not naturally include a size variable for the destination. Gravity equations lacking a size variable for the destination country or region may be problematic for several reasons, among which poor predictions of migration flows if destinations are different in size; or bias in the estimation of the influence on migration of other variables that are correlated with size.

In this paper, we extend the traditional framework by considering destination countries or regions as nests of many underlying atomistic units of choice for migrants, which we

*Department of Agricultural Economics and Rural Development, University of Göttingen, Platz der Göttinger Sieben 5, DE-37073 Göttingen. Liesbeth Colen: liesbeth.colen@uni-goettingen.de Damiaan Persyn: damiaan.persyn@uni-goettingen.de

call ‘opportunities’ following the wording of Docquier *et al.* (2014). An example of an opportunity could be a dwelling or a job. Destinations may have properties of their own such as climate or language, or reflecting properties of the underlying opportunities such as the average wage. Crucially, the number of opportunities appears as the mass or size variable in the gravity equation describing aggregate migration flows. Also higher order moments of the distribution of opportunities are predicted to affect migration flows.

The nested logit model of McFadden (1977) was developed in the field of transportation. We point out several key contributions in the transport and regional science literature which are relevant to economists studying migration, but have been overlooked in recent work. Examples are the constrained models of spatial interaction of Wilson (1967, 1971); the estimation of gravity equations using Poisson regression by Flowerdew and Aitkin (1982); the work of Daly (1982) and Anas (1983) who study aggregation by zones in multinomial models of location choice; the relationship between dummies and ‘balancing constraints’ or ‘multilateral resistance terms’ studied by Fotheringham and Williams (1983); Davies and Guy (1987) and Griffith and Fischer (2013); and lastly Kanaroglou and Ferguson (1996) in an early application of a nested logit model with opportunity sets to migration.

A key result from considering a nested logit structure with a large set of underlying choices is that, under appropriate assumptions, the number of opportunities appears as an attractive factor in the multiplicative gravity equation, serving as the size or mass variable for the destination. The size variable has an associated ‘dissimilarity parameter’ reflecting the uniqueness of the opportunities in the destination. If the unobserved part of individual utility derived from the opportunities within a destination is uncorrelated between them, the dissimilarity parameter equals 1. If the unobserved part of utility is close to perfectly correlated between opportunities for individuals, the dissimilarity parameter tends to 0. Only in this extreme case the number of opportunities drops out of the gravity equation, and we are back in the case where destinations countries or regions can be considered the fundamental unit of choice. We offer several arguments why a coefficient close to 1 is more relevant for applications to migration.

Considering sets of opportunities in the destination also allows to consider some interesting extensions. We highlight two of them: First, if the opportunities are many and can be described stochastically, McFadden (1977) shows that the variance in utility from opportunities becomes an attractive factor of a destinations. This little known property may explain urbanisation trends in spite of high unemployment rates and high living costs in cities. It assumes that migrants are able to gather information about opportunities in the destination which helps them to avoid the less attractive outcomes.

Second, following Daly (1982), if migrants are looking for different types of opportunities (for example jobs and arable land), a single index combining the size of all opportunity sets enters the utility function and the gravity equation as the single mass variable for the destination.

We believe our work presents three contributions. First, we point out the risk associated with considering entire countries or regions as the fundamental unit of choice of migrants, rather than the underlying opportunities contained in them. The RUM model that is traditionally used in migration studies treats destinations as atomistic and does not impose the inclusion of destination size. Many empirical studies fail to include or control for the size of the destination and we point out several studies that may be susceptible to biases as a result. We hope that the gravity equation derived from a nested logit model presented here can help avoiding mistakes in empirical applications. Secondly, our work contributes by linking overlooked, but established and relevant contributions in transportation science to the recent work in economics on migration. Lastly, as far as we know, our analysis is the first empirical application to migration combining multiple size variables in a theoretically consistent fashion.

The remainder of this paper is organised as follows: Section 2 considers the RUM based gravity equation for migration that is popular in current economic research. Section 3 introduces a nested logit model with a countable set of opportunities in each destination, which leads to the appearance of the mass of the destination in the gravity equation. Section 4 considers some important extensions: variance in utility as an attractive force; multiple measures of size of the destination; and the link with models with additional levels of nesting. Section 5 concludes.

2 The traditional approach

Consider the gravity equation for migration as derived by Grogger and Hanson (2011) and many subsequent Contributions (see Beine *et al.*, 2016, for an overview), which is based on random utility maximisation (McFadden, 1974). A potential migrant i in an origin country o compares utility among possible destination countries indexed by $d \in D$, among which the country of origin itself. Following the notation of Beine *et al.* (2016), utility U of individual i is assumed to depend on an index of observables at the destination w_d, and on the bilateral cost c_{od} of moving from o to d:

$$U_{odi} = w_d - c_{od} + \epsilon_{odi}.$$

Assuming that the error term ϵ_{odi} has an iid extreme value distribution results in a convenient expression for the probability P_{od} of an individual in location o to prefer destination $d \in D$ over all other destinations.

$$P(U_{odi} > U_{oxi}) = \frac{exp(w_d - c_{od})}{\sum_{e \in D} exp(w_e - c_{oe})} \qquad \forall \;\; x \in D.$$

If the number of individuals is large, this probability corresponds to the share s_{od} of the population migrating from o to d. Writing pop_o for the number of individuals in o and m_{od} for the number of migrants between o and d:

$$s_{od} \equiv \frac{m_{od}}{pop_o} = \frac{exp(w_d - c_{od})}{\sum_{e \in D} exp(w_e - c_{oe})}. \tag{1}$$

By bringing pop_o to the right hand side and writing y_d for $exp(w_d)$ and $\phi_{od} = exp(-c_{od})$ the resulting expression for the expected number of migrants m_{od} resembles a gravity equation:

$$m_{od} = pop_o y_d \phi_{od} \frac{1}{\sum_e y_e \phi_{oe}}. \tag{2}$$

The population in the origin appears naturally by assuming that there is a given number of individual decision makers in the origin. This number of choice-makers serves as the measure of the mass of the origin. It is perhaps disconcerting that there is no corresponding variable for the size of the destination, as you would expect in a typical gravity equation. Such a destination-size variable may or may not be included by the empirical researcher as a destination-specific explanatory variable in y_d, but its inclusion does not naturally follow from this traditionally used theoretical framework. Frequently the size of the destination is omitted. After adding an appropriately defined multiplicative error term to equation (2) it can be estimated using Poisson pseudo-maximum likelihood. An advantage of the Poisson regression framework is that it is able to handle zero flows and heteroskedasticity, as emphasised by Silva and Tenreyro (2006). Also Flowerdew and Aitkin (1982) discuss the advantages of Poisson regression for the estimation of gravity equations.

As an alternative, one can consider the log-odds, the log of the ratio of two migration shares. To avoid dividing by a small number, one often chooses to divide by the share of stayers (individuals in o who prefer their current country of residence o over all possible destinations)

$$s_{oo} = \frac{m_{oo}}{pop_o} = \frac{exp(w_o - c_{oo})}{\sum_{d \in D} exp(w_d - c_{od})}. \tag{3}$$

Taking the log of the ratio of the shares (1) and (3) results in a linear function of the difference in the variables determining the attractiveness of a location.

$$\ln\left(\frac{s_{od}}{s_{oo}}\right) = \ln\left(\frac{m_{od}}{m_{oo}}\right) = w_d - w_o - (c_{od} - c_{oo}) \tag{4}$$

Again the researcher is left to decide whether the vector of explanatory variables should include some measure of size, which would logically then be included in both in w_d and w_o. This equation can be estimated by OLS. Advantages of this functional form are that it avoids numerical optimization and allows for instrumental variable estimation (Berry, 1994).

The omission of a mass variable for the destination can be problematic in empirical applications. Consider an analysis based on equation (1) or (4) for the case of just two origin and destination countries of vastly different size but with similar wages. An example could be Malta and Italy. Consider the case where size is not controlled for, wages are equal to 1 in both countries, and assume $c_{oo} = 0$ and $c_{od} = 4.6$ for $o \neq d$. For these values the model predicts 1 percent of inhabitants in either country will migrate. Malta has a population of about 500.000, so 5.000 Maltese would be predicted to migrate to Italy. Italy has a population of about 60 million, so 600.000 Italians would be predicted to migrate to Malta, more than doubling its population. This is obviously not realistic. The model does not reflect our intuition that migration flows also depend on the size of the destination. In real data the share of individuals choosing a destination likely depends on its size (as we will argue formally in the next section). An analysis on real-world data ignoring size is likely to find unexpectedly large residual flows to large destinations.

Such a pattern may be reflected in the results of Grogger and Hanson (2011, p. 54) who omit measures for the size of the destination in their analysis of international migration. Among all destinations considered they observe the largest residual attractiveness for the USA and Germany. Offered explanations are higher wages in these countries, labour-recruitment strategies in the 1960s, post-war asylum policies and immigrant networks. Whereas such factors may indeed play a role, a more basic explanation for the large residual migration flows to these countries is that these are the largest destination countries in an analysis that does not control for size.

Ortega and Peri (2013) and Beine *et al.* (2019), estimate a dynamic version of (4) which may be stylised as

$$\ln(m_{odt}) = \ln(m_{oot}) + w_{dt} - w_{ot} - c_{odt} + \xi_{odt}.$$

Beine *et al.* (2019) emphasise the importance of including origin-time fixed effects to control for m_{oot}. These origin-time fixed effects would also capture the effect of any time-varying size variable in w_{ot}. But there is an asymmetry in that no destination-time fixed effects are included, such that any time-varying effect of the size of the destination in w_{dt} (such as the number of jobs or the GDP which may be important given the focus on the business cycle in their analysis) would not be controlled for.

3 Putting back size in the RUM based gravity equation for migration

The previous section showed that the RUM based gravity equation for migration commonly used in the economics literature leads to a gravity equation which does not naturally include a size variable for the destination, and that this may be problematic in applied work if a size variable is not explicitly added (or if insufficient dummies are included, or if dummies are included but subsequently are interpreted). This section introduces a countable number of opportunities in the destination, and shows how under given assumptions their number naturally appears in the gravity equation for aggregate migration flows. This idea is similar to the arguments put forward by Docquier *et al.* (2014) from whom we borrow the term 'opportunities'. We formalise the concept here drawing from a large and old literature on discrete choice in transportation and geography (see for example McFadden, 1977; Flowerdew and Aitkin, 1982; Daly, 1982; Anas, 1983). We summarise several of the models from this literature with an application on migration in mind.

3.1 Aggregating opportunities

Assume now that migrants do not seek countries but rather some type of opportunity, for example a dwelling or a job, contained in them. To fix ideas, these opportunities may be thought of as pure rival goods: a dwelling can be occupied by only a certain number of inhabitants, excluding others. This contrasts with other determinants of migration that are frequently considered such as climate, the average wage or a common colonial history. Such variables are clearly important to migrants and should be controlled for, but they have no rival quality and we cannot assign a number, size or mass to them.

Following Anas (1983) we first consider the case where utility is not correlated between opportunities. In this case, the equations (2) and (3) remain valid, with the adjustment that the choice now is no longer over destination countries, but over opportunities, which we index by $f \in F_d$ (for felicity), with F_d the set of opportunities in country d with cardinality N_d, such that the share of individuals in origin o choosing opportunity f in destination d equals

$$s_{ofd} \equiv \frac{m_{ofd}}{pop_o} = \frac{exp(w_{fd} - c_{ofd})}{\sum_{e \in D} \sum_{g \in F_e} exp(w_{ge} - c_{oge})}$$

If we are interested in the number of migrants to countries we simply have to add the probabilities or shares corresponding to the opportunities contained within each destination country. Consider the case where the utility derived from the different opportunities within a country is identical such that $w_{fd} = w_d$ and $c_{ofd} = c_{od}$:

$$\begin{aligned} s_{od} &= \sum_{f \in F_d} \frac{m_{ofd}}{pop_o} = \sum_{f \in F_d} \frac{exp(w_{fd} - c_{ofd})}{\sum_{e \in D} \sum_{g \in F_e} exp(w_{ge} - c_{oge})} \\ &= \frac{N_d exp(w_d - c_{od})}{\sum_{e \in D} N_e exp(w_e - c_{oe})} \end{aligned} \tag{5}$$

Written as a gravity equation and writing y_d for $exp(w_d)$ and $\phi_{od} = exp(-c_{od})$ as before, we obtain

$$m_{od} = pop_o N_d y_d \phi_{od} \frac{1}{\sum_{e \in D} N_e y_e \phi_{oe}} \tag{6}$$

Considering countable, independent, atomistic units of choice (opportunities) within destinations, leads the size of the opportunity set in the destination to appear naturally in the gravity equation.

The log-odds are

$$\ln\left(\frac{s_{od}}{s_{oo}}\right) = \ln\left(\frac{m_{od}}{m_{oo}}\right) = \ln(N_d) - \ln(N_o) + w_d - w_o - (c_{od} - c_{oo}).$$

Whereas the number of potential migrants in the origin pop_o cancels out when considering log-odds, the log of a size variable proxying the number of opportunities should be included for both origin and destination. The expected coefficient on these size variables is 1 and -1 given the extreme assumption of strictly uncorrelated opportunities.

3.2 Correlated opportunities: nested logit

It may be unrealistic to assume that the unobserved part in the utility derived from opportunities within the same country is uncorrelated. McFadden (1977) considers a more general utility function with correlated opportunities (see also Train (2002) and

Kanaroglou and Ferguson (1996) for an early application to migration). Without loss of generality, the utility derived from choosing a certain opportunity f within a destination (country, region,...) d is decomposed in a part w_d that is common among opportunities within d, and a part z_{fd} specific to the opportunity. Following Cardell (1997); Berry (1994) and Bertoli and Fernández-Huertas Moraga (2015) also the unobserved part of utility is split in a destination and opportunity-specific part, such that

$$U_{ofi} = w_d + z_{fd} - c_{od} + (1 - \lambda_d)\xi_{di} + \lambda\epsilon_{ofi}, \tag{7}$$

ξ_{di} is distributed iid extreme value and ϵ_{ofi} is distributed as the unique random variable that ensures that also the joint error term $(1 - \lambda_d)\xi_{di} + \lambda\epsilon_{ofi}$ is iid extreme value. The parameter λ_d or 'dissimilarity parameter' governs the correlation between the unobserved part of utility for individuals between opportunities within destinations.

We consider the following convenient decomposition of the corresponding probability that an alternative f within the opportunity set F_d of destination d is chosen:

$$\begin{aligned} P_{o,f} &= P_{f|F_d} \cdot P_{o,F_d} \\ P_{o,F_d} &= \frac{\exp(w_d - c_{od} + \lambda_d I_d)}{\sum_e exp(w_e - c_{o,e} + \lambda_e I_e)} \\ P_{f|F_d} &= \frac{\exp(z_{fd}/\lambda_d)}{\sum_{g \in F_d} exp(z_{gd}/\lambda_d)} \\ I_d &= \log \sum_{g \in F_d} \exp(z_{gd}/\lambda_d). \end{aligned}$$

If interest lies with predicting migration flows to destinations d (countries, regions, cities,...) which nest opportunities, rather than which opportunity is chosen within them, then only the aggregate level flow described by P_{o,F_d} and the log-sum or inclusive value term I_d are relevant. Under the assumption that the deterministic part of opportunity-specific utility is constant within destination countries ($z_{fd} = z_d$), it holds that $I_d = \log(N_d) + z_d/\lambda_d$ and

$$P_{o,F_d} = \frac{m_{od}}{pop_o} = \frac{exp(w_d + z_d + \lambda_d \log N_d)}{\sum_e exp(w_e + z_e + \lambda_d \log N_e)}$$

or writing y_d for $exp(w_d)$ and $q_d = exp(z_d)$ and $\phi_{od} = exp(-c_{od})$ as before, it obtains that

$$m_{od} = pop_o y_d q_d N_d^{\lambda_d} \phi_{od} \frac{1}{\sum_e y_e q_e N_e^{\lambda_d} \phi_{od}}.$$

Here y_d collects the influence of variables pertaining the country (climate, etc.), q_d pertains to characteristics of the opportunities (average wage, housing price level, etc.), N_d is the number or mass of opportunities (number of jobs, houses, arable land area, etc.), and the associated parameters $0 \leq \lambda_d \leq 1$ reflects how independent the unobserved part of utility is between opportunities in each destination. The log-odds at the country level then are given by

$$\ln\left(\frac{s_{od}}{s_{oo}}\right) = \ln\left(\frac{m_{od}}{m_{oo}}\right) = \lambda_d \ln(N_d) - \lambda_d \ln(N_o) + w_d - w_o + z_d - z_o - (c_{od} - c_{oo}).$$

One may obviously choose to test or impose the assumption that the dissimilarity between opportunities is equal among destinations ($\lambda_d = \lambda$). If the correlation in the unobserved part of utility within destinations is large, opportunities within destinations are perceived as similar and the associated dissimilarity parameters λ_d will be small. If the model includes most relevant control variables, considers other levels of nesting that group destination countries or regions with similar properties (language, ethnicity), then the residual unobserved component in utility (7) will be small, and opportunities will be perceived as dissimilar.

These results suggest that an applied researcher — even if uninterested in the concept of underlying opportunities or the mathematics of nested logit models — would be well advised to include some measure of size of the destination in multiplicative gravity equations, or the log of the size of both origin and destination in log-odds regressions. These size variables should proxy the size of the opportunity sets in the locations. The associated coefficients serves as an inverse indicator of correlation between opportunities within destinations. A coefficient significantly smaller than 1 may suggest that the size proxy is not appropriate, that relevant control variables are missing, or further nesting of destinations should be modelled to reduce unobserved correlation within destinations.

4 Extensions

The previous sections argued that the presence of a countable or measurable opportunity set in the destination requires the introduction of a size variable in the gravity equation for migration. Introducing such an opportunity set allows to consider other issues as well, however. We first consider the case with stochastic opportunities where, perhaps surprisingly, the variance in utility of opportunities can be shown to be an attractive factor of a destination. Second, we consider the case of different migrants looking for different types of opportunities, and the nonlinearity this introduces in the gravity equation. Lastly, we consider the case of higher-level nesting.

4.1 Heterogeneous opportunities within destinations

If the observed part of utility associated with the opportunities within each destination can be described stochastically and is approximately iid normally distributed with mean z_d and variance ω_d, the utility of choosing destination d equals[1]

$$P_{F_d} = \frac{\exp(w_d + z_d + \frac{\omega_d}{2\lambda} + \lambda \log(N_d))}{\sum_e \exp(w_e + z_e + \frac{\omega_e}{2\lambda} + \lambda \log(N_e))}.$$

Such that, perhaps surprisingly, underlying variability in the utility derived from opportunities makes a destination more attractive. This assumes that a migrant is able to observe and choose the opportunity within the destination. This assumption may be appropriate in applications with good information flows. One relevant case could be migration over relatively small distances, between regions within a relatively developed country, and not in situations of deprivation, where migration often occurs only after a jobmarket match has been made. Another example could be migrant networks passing on information on specific available jobs and other opportunities to prospective migrants in the home country. The attractive effect of the variance in opportunities may offer an explanation for urbanisation trends in developing countries. Cities with a wide variation in opportunities are predicted to be attractive, in spite of high unemployment and other factors suggesting a low average expected return should a migrant arrive uninformed and unconnected. It can also offer an explanation on why countries with high income dispersion are found be attractive in empirical studies on international migration.

[1] This section assumes $\lambda > 0$, McFadden (1977) considers the the limiting case $\lambda \to 0$ where the probability almost surely converges to

$$P_{F_d} \underset{\text{a.s.}}{\to} \frac{exp(w_d + \max_f z_{fd})}{\sum_e exp(w_e + \max_f z_{fe})}.$$

When opportunities are perceived as extremely similar conditional on w_d and z_{df}, their number becomes irrelevant and (apart from destination specific factors in w_d) only the maximum attainable z_{df} within each destination matters for the probability of destination d to be chosen, and the properties z_{fd} of this best opportunity within d can be absorbed in the destination specific variables contained in w_d. This case does not seem relevant for applications to migration.

4.2 Multiple measures of destination size

Imagine that migrants are heterogeneous and have different motivations for migration. In this case it may be unclear what the size variable should be, e.g. number of jobs, housing, etc. As argued by Daly (1982), different size variables must then be included in a single index weighted with their respective dissimilarity parameters, and enter the gravity equation in a non-linear fashion. With different relevant size variables in the destination, $N_{1d}, N_{2d}, \ldots$ the utility from choosing destination d equals

$$w_d + z_d + \lambda \log(N_{1d} + \lambda_2 N_{2d} + \ldots).$$

In an application studying rural-rural and rural-urban migration a researcher may want to include both a measure of arable land and the number of jobs (or local GDP) to proxy the attractiveness of rural and urban destinations. But the above result suggests that one cannot introduce these size variables additively in a log-linear gravity equation. One should rather consider the above specific functional form. The non-linearity in the parameters introduced when considering multiple size variables complicates maximum likelihood estimation and implies that the log-odds are no longer estimable by OLS.

4.3 Spatial equilibrium, urbanisation and the mass variable in the gravity equation

The previous section argued that in an ideally specified discrete choice model residual destination-specific correlation in utility should be small, and the parameter associated with destination size should be close to 1. This section considers aggregation and spatial equilibria as two more reasons why a coefficient close to 1 is reasonable base-case to expect.

Consider the case of $r+1$ symmetric regions, with identical attributes $y_k q_k = 1$, populations $pop_k = pop$, and $N_k = N$ opportunities. Individuals face 0 migration costs when choosing their current region $\phi_{kk} = exp(0) = 1$ and $\phi_{kl}|_{k \neq l} = \phi$ (with $0 < \phi < 1$) when migrating. The migration flow from k to l then is given by

$$m_{lk}|_{k \neq l} = popN^{\lambda}\phi \frac{1}{N^{\lambda} + rN^{\lambda}\phi}$$

Now consider a scenario where r of the locations are considered as a single destination (say r locations are Italian) by the inhabitants of the remaining location (Malta). From the point of view of Malta the expected migration flow when adding up the individual flows to all r Italian regions is

$$r \cdot popN^{\lambda}\phi\frac{1}{N^{\lambda}+rN^{\lambda}\phi} \tag{9}$$

or when considering Italy as a single destination of size (rN)

$$pop(rN)^{\lambda}\phi\frac{1}{N^{\lambda}+(rN)^{\lambda}\phi} \tag{10}$$

An obvious sufficient condition for these two perspectives to result in the same predicted number of migrants is $\lambda = 1$. This is the point of Daly (1982) on requiring $\lambda = 1$ for migration to be proportional to the destination size, and for the gravity equation to be neutral to the level of aggregation of destinations. This perfect scaling with size fails if opportunities are seen as closer substitutes within locations compared to across.

A further case for $\lambda = 1$ can be made by considering a spatial equilibrium. It is useful here to consider r as a measure of aggregate size. In the example Italy is r times larger than Malta since it contains r localities each of size N whereas Malta consists of only one such locality. Consider the reverse flow, from all of the r Italian locations to Malta. Continuing to assume that there are costs to migration between the r Italian locations, the total migration flow from Italy to Malta is given by the sum of the flows from the individual r Italian locations to location $r + 1$, Malta:

$$r \cdot popN^{\lambda}\phi\frac{1}{N^{\lambda}+rN^{\lambda}\phi}$$

which is the same as expression (9). A spatial equilibrium exists when this flow equals the flow in the opposite direction given by equation (10), or

$$r \cdot popN^{\lambda}\phi\frac{1}{N^{\lambda}+rN^{\lambda}\phi} = pop(rN)^{\lambda}\phi\frac{1}{N^{\lambda}+(rN)^{\lambda}\phi}$$

or

$$r\frac{1}{N^{\lambda}+rN^{\lambda}\phi} = r^{\lambda}\frac{1}{N^{\lambda}+(rN)^{\lambda}\phi}.$$

For $\lambda = 0$ the equation simplifies to $r = 1$: In case destinations are considered as atomistic as in the traditional analysis described in section 2, migration flows are symmetric only if locations are of equal size. If we assume that opportunities equal population, this implies that the only stable spatial distribution of population is one with an equal population in both aggregate location, Italy and Malta.

For $\lambda = 1$, in contrast, the equation holds for any r: for any size difference r the predicted migration flows in both directions will be equal, implying that any initial size difference is a stable spatial equilibrium. This is of course closer to reality: there tends to be no net migration between countries with similar properties (average wage, etc.) even if they are of very different size.

The arguments in the previous section on the interpretation of the dissimilarity parameter together with those presented here on the conditions for scalability and spatial equilibria show that a reasonable value to expect for the mass variable in a well specified gravity equation is close to 1. A coefficient of 0 would rather imply that population spreads out equally among all destinations.

When estimating a gravity equation for migration in China from rural locations to cities Xing and Zhang (2017) find size coefficients close to 1.[2] However, they conclude from this that 'migrants derive higher utilities from larger cities' and that this explains the growth of larger cities. This seems unfounded as we showed that a coefficient of 1 on a size variable rather corresponds to the base case where migration flows do not alter a given spatial population distribution, e.g. a situation without any urbanisation trend.

4.4 Some remarks on the relation between discrete choice, constrained gravity equations and multilateral resistance terms

There is an asymmetry in how the size of the origin (the number of choice-making agents pop_o) and the size of the destination (the number of opportunities N_d) enter the gravity equation (6): whereas N_d appears in the nominator and denominator, pop_o only shows up in the nominator but not in the denominator. This stems from an asymmetry in the assumptions: The number of agents in each origin is given or fixed. Therefore if less individuals choose a specific destination, some other destination (or the origin) must experience a higher inflow from this origin. The number of arrivals in each destination is not fixed, in contrast. If less migrants choose a specific destination, there typically is no constraint enforcing that the decrease in inflows from one origin must be compensated by an inflow from another origin. Given that only the number of potential migrants in each origin is fixed, this model is known as the origin, production, or single constrained gravity equation (see Wilson, 1971). Taking the number of decision takers in each origin as given, but not the inflow per destination, seems particularly warranted in the study of 'supply driven' phenomena such as refugee flows or migration from underdeveloped

[2]They find estimates below and above 1. Their preferred estimate is 1.056 with a standard error of 0.133

countries where a decrease in the inflow from one origin to a specific destination does not imply an increase in the inflow from another origin.

The assumption of a fixed number of individual choice-makers in the origin is embedded in discrete choice frameworks such as multinomial logit and nested logit models. It can be implemented in a Poisson regression by including origin fixed effects, as emphasised by Fally (2015) in the context of international trade, but was already known by for example Fotheringham and Williams (1983); Davies and Guy (1987) and Griffith and Fischer (2013). The factor $1/\sum_{e\in D} N_e y_e \phi_{oe}$ in equation (6) assures that the constraint holds that $\sum_o m_{od} = pop_o$. It is therefore called a 'balancing factor' by Wilson (1971), and corresponds to one of the 'multilateral resistance' terms of Anderson and Wincoop (2003) or Bertoli and Fernández-Huertas Moraga (2013).

In the context of regional migration, or migration between similar countries, it may be reasonable to also consider the total inflow in each destination as given. An example would be the case where opportunities are jobs that need to be filled. If an inflow from one origin to some destination decreases, the jobs in the destination will be filled by an increase in the flows from other origins. Strictly imposing this constraint gives rise to the Wilson doubly constrained model (see Wilson, 1971) which is isomorphic to the gravity model of Anderson and Wincoop (2003).[3] The doubly constrained model can be empirically implemented in a discrete choice framework by including destination specific constants. As the number of choice makers is fixed inherently in discrete choice models, the origin-constraint always holds[4]. Using a Poisson regression, the doubly constrained model is what is estimated when including origin and destination dummies (fixed effects). The estimated values of these dummies corresponds to the origin and destination 'balancing constraints' or 'multilateral resistance terms'.

This text focussed on the origin-constrained model which has received more attention in the recent economic literature on migration. However, although most studies derive the origin-constrained model from a discrete choice framework, many studies are subsequently – perhaps unknowingly – estimating a doubly-constrained model by including both origin and destination fixed effects in the Poisson regressions in their empirical implementation.

[3]Whereas Anderson and Wincoop (2003) derived their 'doubly constrained' model using CES preferences, Wilson (1970, 1971) used information theory (entropy maximisation) and Anas (1983) used discrete choice theory. See for example Persyn and Torfs (2016) for an application of a CES based doubly constrained model to commuting.

[4]Anas (1983) shows that the maximum likelihood estimate of the destination specific constant in a multinomial framework equals the expression for the balancing constraint (multilateral resistance term).

5 Summary and Conclusion

This paper suggested a nested random utility framework for migration where destinations consist of large sets of opportunities. The model servers as an extension or alternative to the prevalent specifications found in the study of migration in the economics literature where destination countries or regions are considered as the fundamental unit of choice of migrants, even if they differ significantly in size. If the opportunities are equally valuable to migrants and uncorrelated, their number enters as an attractive factor for the destination in the multiplicative gravity equation describing aggregate flows. If the unobserved part of utility is correlated among opportunities, the size variable in the aggregate gravity equation for migration is has an associated coefficient smaller than one, which attenuates the effect of size depending on the correlation between opportunities. The traditional gravity equation where countries are the relevant unit of choice for migrants is obtained as a limiting case with perfectly correlated opportunities. In this case only properties at the country level which are unrelated to size, such as climate, average wage, or the unemployment rate, explain migration flows. We show, however, that this case is unlikely to be relevant, as it leads to predictions that are clearly violated in the data, such as an equal spatial distribution of population among all locations.

If the deterministic part of utility derived from opportunities differs between them, but can be described statistically, the variability in opportunities appears as an attractive factor of the destination, divided by the parameter expressing correlation. Estimation is complicated by this resulting non-linearity. This result assumes that migrants can choose between opportunities at the destination, ignoring less favourable ones. This is only realistic if some flow of information exists to prospective migrants and other assumptions hold. In this case, destinations with equal average opportunities but more extremes opportunities are more attractive. Such mechanisms may be behind the finding that cities and countries with more unequal outcomes and less redistribution are surprisingly attractive.

The main conclusion of this paper is not that applied researchers should consider a complete nested discrete choice structure considering disaggregate opportunities within destinations. Applied researchers should be aware, however, that treating countries or regions as atomistic and mass-less may be a form of model misspecification leading to biased inference and prediction error. A model including a basic proxy for the size of the set of available choices in each destination, with the correct (intuitive) functional form as described in the paper, is bound to perform much better than the alternative omitting any such variable.

A researcher should not be surprised to see a strong correlation between country size and residuals (or estimated destination country dummies) in analyses omitting a size variable. The finding of, for example, Grogger and Hanson (2011) and Xing and Zhang (2017) that larger destinations attract more migrants is to be expected: a coefficient of one associated with size in the main analysis (or in a subsequent analysis of residuals or dummies) rather suggest the base-case of a stable spatial equilibrium, where the population share of each location would remain constant. It does not reflect a trend towards urbanisation or excess migration flows to large destinations.

References

Anas, A. (1983). Discrete choice theory, information theory and the multinomial logit and gravity models. *Transportation Research Part B: Methodological*, **17** (1), 13–23.

Anderson, J. E. and Wincoop, E. V. (2003). Gravity with Gravitas: A Solution to the Border Puzzle. *The American Economic Review*, **93** (1), 23.

Beine, M., Bertoli, S. and Fernández-Huertas Moraga, J. (2016). A Practitioners' Guide to Gravity Models of International Migration. *The World Economy*, **39** (4), 496–512.

—, Bourgeon, P. and Bricongne, J. (2019). Aggregate Fluctuations and International Migration. *The Scandinavian Journal of Economics*, **121** (1), 117–152.

Berry, S. T. (1994). Estimating Discrete-Choice Models of Product Differentiation. *The RAND Journal of Economics*, **25** (2), 242–262.

Bertoli, S. and Fernández-Huertas Moraga, J. (2013). Multilateral resistance to migration. *Journal of Development Economics*, **102**, 79–100.

— and — (2015). The size of the cliff at the border. *Regional Science and Urban Economics*, **51**, 1–6.

Cardell, N. S. (1997). Variance Components Structures for the Extreme-Value and Logistic Distributions with Application to Models of Heterogeneity. *Econometric Theory*, **13** (2), 185–213.

Daly, A. (1982). Estimating choice models containing attraction variables. *Transportation Research Part B: Methodological*, **16** (1), 5–15.

Davies, R. B. and Guy, C. M. (1987). The Statistical Modeling of Flow Data When the Poisson Assumption Is Violated. *Geographical Analysis*, **19** (4), 300–314.

Docquier, F., Peri, G. and Ruyssen, I. (2014). The Cross-country Determinants of Potential and Actual Migration. *International Migration Review*, **48** (1_suppl), 37–99.

Fally, T. (2015). Structural gravity and fixed effects. *Journal of International Economics*, **97** (1), 76–85.

Flowerdew, R. and Aitkin, M. (1982). A Method of Fitting the Gravity Model Based on the Poisson Distribution. *Journal of Regional Science*, **22** (2), 191–202.

Fotheringham, A. S. and Williams, P. A. (1983). Further Discussion on the Poisson Interaction Model. *Geographical Analysis*, **15** (4), 343–347.

Griffith, D. A. and Fischer, M. M. (2013). Constrained variants of the gravity model and spatial dependence: model specification and estimation issues. *Journal of Geographical Systems*, **15** (3), 291–317.

Grogger, J. and Hanson, G. H. (2011). Income maximization and the selection and sorting of international migrants. *Journal of Development Economics*, **95** (1), 42–57.

Kanaroglou, P. S. and Ferguson, M. R. (1996). Discrete spatial choice models for aggregate destinations. *Journal of Regional Science*, **36** (2), 271–290.

McFadden, D. (1974). The measurement of urban travel demand. *Journal of Public Economics*, **3** (4), 303–328.

— (1977). Modelling the Choice of Residential Location. *Cowles Foundation Discussion Papers*, (477).

Ortega, F. and Peri, G. (2013). The Role of Income and Immigration Policies in Attracting International Migrants. *Migration Studies*, **1** (1), 47–74.

Persyn, D. and Torfs, W. (2016). A gravity equation for commuting with an application to estimating regional border effects in Belgium. *Journal of Economic Geography*, **16** (1), 155–175.

Silva, J. M. C. S. and Tenreyro, S. (2006). The Log of Gravity. *Review of Economics and Statistics*, **88** (4), 641–658.

Train, K. (2002). *Discrete Choice Methods with Simulation*. Cambridge University Press.

Wilson, A. (1967). A statistical theory of spatial distribution models. *Transportation Research*, **1** (3), 253–269.

— (1970). *Entropy in urban and regional modelling*. London: Pion.

— (1971). A family of spatial interaction models, and associated developments. *Environment and Planning*, **3**, 32.

Xing, C. and Zhang, J. (2017). The preference for larger cities in China: Evidence from rural-urban migrants. *China Economic Review*, **43**, 72–90.